Introduction to Bayesian Networks

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@icahnInstitute UCLA workshop, July, 2013---Jun Zhu, Ph. D.
Outline

1. What are Bayesian networks?
2. What are usages of Bayesian networks?
3. What is a naïve Bayes net?
4. How to train a Bayesian network?
5. How to construct a Bayesian network?
6. What is different in biology system for BN?
7. What is Dynamic BN?
What are Bayesian networks?

- A Bayesian network is an expert system that captures all existing knowledge;
- They are also called belief networks, Bayesian belief networks, causal probabilistic networks;

- A Bayesian network consists of
  - a directed acyclic graph (a set of nodes and directed edges connecting nodes)--DAG
  - A set of conditional probability tables (for discrete data) or probability density functions (for continuous data)
A Bayesian network

DAG

Conditional probability tables

\[ p(C \mid A, B) \]
\[ p(D \mid B) \]
\[ p(E \mid B) \]
\[ p(F \mid C) \]
Bayesian network

- A tree is a Bayesian network
Bayesian network

- A Bayesian network is not a tree
Bayesian network

- Conventional Notations

\[ p(A) = \prod_{i} p(A_i \mid pa(A_i)) \]

\( A = \{A_1, A_2, \ldots, A_n\} \) are nodes.

\( p(A) \) is the joint probability of nodes \( A \).

\( pa(A_i) \) are parent nodes of \( A_i \).
Bayesian network

- A diverging structure

out-degree = 4
Bayesian network

- A converging structure

\[ \text{in-degree} = 3 \]
Bayesian network

- Why a DAG is required?

\[ p(A) = \prod_{i} p(A_i \mid pa(A_i)) \]

- It is guaranteed that there is a node \( A_j \) in a DAG that has no child.

\[ p(A) = p(A \setminus \{A_j\}) p(A_j \mid A \setminus \{A_j\}) \]
\[ = p(A \setminus \{A_j\}) p(A_j \mid pa(A_j)) \]
\[ = \left(\prod_{i \neq j} p(A_i \mid pa(A_i))\right) \ast p(A_j \mid pa(A_j)) \]
Bayesian network: usages

- Bayesian networks can be used to predict outcomes or diagnose causal effects (if structures are known)

- Bayesian networks can be used to discover causal relationships (if structures are not known)
Bayesian network: an example

- A burglar alarm system
Bayesian network: a classifier

- What is a naïve Bayes net

\[
p(A, B, C, D, E) = p(B \mid A)p(C \mid A)p(D \mid A)p(E \mid A)
\]

\[
p(A \mid B, C, D) = \frac{p(A, B, C, D, E)}{p(B, C, D, E)}
\]
Bayesian network

- How to train a Bayesian network

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Bayesian network

- How to construct a Bayesian network? Enumerating possible structures
Bayesian network

- How to construct a Bayesian network? Enumerating all possible structures is impossible

\[ N^N, \text{ } N \text{ is the number of nodes} \]
Bayesian network

• How to construct a Bayesian network? Heuristic approach
Bayesian network

• How to construct a Bayesian network? Heuristic approach

Parameters to estimate=3x3x3

Parameters to estimate=3x3x3x3
Bayesian network

- How to construct a Bayesian network? Heuristic approach

\[
p(M \mid D) = \frac{p(D \mid M) p(M)}{p(D)}
\]

\[-2 \ln(p(M \mid D)) \approx \text{BIC} = -2 \ln(D \mid \hat{M}) + k \ln(n)\]

\(n\) : number of samples

\(k\) : number of parameters to estimate
Bayesian network

- How to construct a Bayesian network? averaging

Zhu et al., PLoS CompBio, 2007
Zhu et al., Nature Genetics, 2008
Bayesian network

- How to construct a Bayesian network? Enforcing DAG after averaging
  1. Calculate shortest distance
  2. Identify loops
  3. Remove the weakest link in a loop
  4. Go to step 1
Bayesian network

- How to construct a Bayesian network? Upper limit on in-degree

Parameters to estimate = $3^{n+1}$
Bayesian network

- Continuous vs discrete models
- Discrete model is faster, easier to capture high order interactions
- Any discretization lost information

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- Missing information
Bayesian network

• Feedbacks in biological systems

\[ \frac{dg}{dt} = \frac{\theta^n}{\theta^n + c^n} - \alpha g = \mu(g, c) \]
\[ \frac{dc}{dt} = \beta g - \delta c = \nu(g, c) \]
Bayesian network

- **Feedbacks in biological systems**

Changing parameters in activation results in negative correlation.
Bayesian network

- Feedbacks in biological systems

Changing parameters in inhibition results in positive correlation.
Bayesian network

- Biological network is context specific
- Bayesian network is just a snapshot under a specific condition
Bayesian network

- Dynamic Bayesian Bayesian network?

Dynamic Bayesian network

Granger causality

Zhu et al., PLoS CompBio, 2010
Bayesian network

- Granger’s causality test

\[ y_{n,t} = \gamma_n y_{n,t-1} + \varepsilon_{n,t} \]
\[ y_{n,t} = \alpha_n y_{n,t-1} + \beta_n x_{n,t-1} + \mu_{n,t} \]

Zhu et al., PLoS CompBio, 2010
Bayesian network

- Dynamic Bayesian network

\[
p(X_1, \ldots, X_T) = \prod_{t=1}^{T} \prod_{i} p(X_t^i | \text{Pa}(X_t^i))
\]

\[
\text{Pa}(X_t^i) \in X_{t-1} \cup X_t
\]

Zhu et al., PLoS CompBio, 2010
Dynamic Bayesian network: Assumptions

1. Sampling time is faster than reaction time
2. Sampling time is the exact same as the reaction time
3. Sample time is slower than reaction time

Zhu et al., PLoS CompBio, 2010
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